Int. J. Heat Mass Transfer. Vol. 16, pp. 1803–1805. Pergamon Press 1973. Printed in Great Britain

SHORTER COMMUNICATIONS

NOTE ON TANGENTIAL EDDY DIFFUSIVITY IN A CIRCULAR TUBE

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(*Received* 18 *December* 1972)

IN THEIR paper on measurements of the radial and tangential eddy diffusivities of heat and mass in turbulent flow in a plain tube, Quarmby and Quirk [1] state that, within their knowledge, "there are no published theories of turbulent exchange, whether using a spherical eddy model or otherwise, which give the ratio of the tangential and radial diffusivities of heat or mass". The reader who is not thoroughly familiar with the field of turbulent transport properties might conclude from this statement that actually such theories do not exist yet. However, this is not the case. The objective of this note is to shortly review the available theoretical approaches for predicting anisotropic eddy diffusivities by Bobkov *et al.* [2, 3] and Ramm and Johannsen [4–6] as well as to compare their results for the circular tube with the experimental findings of Quarmby and Quirk [1].

Bobkov *et al.* [2, 3] performed a combined theoretical and experimental approach to predict velocity distribution and turbulent transport properties in fully-developed channel flow. In case of thermal diffusivity, it involves an experimental investigation of the statistical characteristics of temperature fluctuations and a subsequent analysis applying the theory of homogeneous diffusion for limited regions of the flow, i.e. where the assumptions of homogeneous diffusion hold. Their results for the ratio of the tangential and radial diffusivity for heat, $\varepsilon_{h\omega}/\varepsilon_{hr}$, for liquid metal flow through a circular tube have been presented in [2] in graphical form as well as by the approximate equation,

$$\frac{\varepsilon_{h\omega}}{\varepsilon_{hr}} = 1 + \frac{0{\cdot}2}{1{\cdot}02 - z}$$

where z is the dimensionless tube radius. It may be noted that both presentations do not exhibit any dependence on Reynolds number, though the authors state that a weak effect of Reynolds number has been found in their results. Further applications of the method to axisymmetric flow through channels of noncircular cross section have been reported in [3].

The theoretical approach of Ramm and Johannsen [4, 5] may be considered as a refined "zero-equation statistical

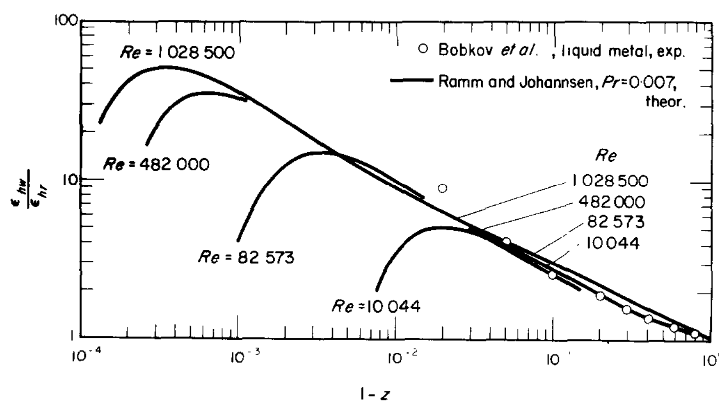

FIG. 1. Ratio of eddy diffusivity for heat in tangential direction to that in radial direction vs relative wall distance.

turbulence model". It is based on the principal ideas underlying Buleev's model of turbulent transfer in three-dimensional fluid flow [7, 8] which permits determination of all of the components of the turbulent shear stress tensor as well as the turbulent heat fluxes independently of each other. This model has been substantially improved and extended to remove certain deficiencies which become apparent on physical grounds and with respect to experimental evidence [4, 5, 9] in predicting turbulent transport properties. From this theory, the ratio $\varepsilon_{h\omega}/\varepsilon_{hr}$ for fully-developed tube flow has been found [4] to vary across the flow section from unity in the center to rather high values in the region close to the wall, where a maximum seems to exist (Fig. 1). As Reynolds number increases, the value of the maximum rises rapidly and its location approaches the wall. With respect to Prandtl number, it was found that its effect is of rather minor importance in the range of $0.005 \leqslant Pr \leqslant 10$. It may be mentioned that very similar results have been obtained for fully-developed axial turbulent flow through concentric annuli [4] and regular rod arrays [5, 6]. The results for the circular tube are in excellent agreement with those of Bobkov *et al.* [2] in the ranges of $z < 0.95$ and $Re \lesssim 10^5$ (Fig. 1).

In Fig. 2, the data of Quarmby and Quirk [1] for both heat and mass transfer at a Prandtl number of 0·71 and a Schmidt number of 0·77, respectively, are compared with the prediction of Ramm and Johannsen for heat transfer ($Pr = 0.7$). In view of the scatter of the data, the agreement is satisfactory. The predictions again indicate an increase of the eddy diffusivity ratio with Reynolds number as far as the wall near region is concerned. This effect, however, seems to fall well within the limits of experimental error and is thus not evident in the measurements.

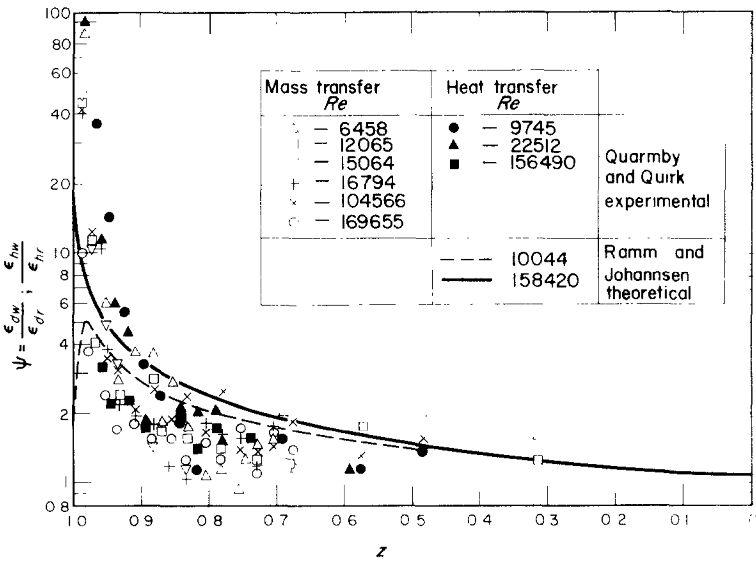

FIG. 2. Ratio of mass and heat eddy diffusivities in tangential direction to that in radial direction vs relative radius.

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Int. J. Heat Mass Transfer. Vol. 16, pp. 1805–1807. Pergamon Press 1973. Printed in Great Britain

APPROXIMATE METHOD FOR ABSORPTION AT HIGH TEMPERATURES

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(*Received* 12 *December* 1972 *and in revised form* 8 *March* 1973)

IT WAS shown in a previous study that at elevated temperatures the absorption of a radiating gas is significantly affected by overlapping "hot bands" [1]. The final theoretical relations are, however, complex which complicates the calculations. Furthermore, the extension of the results to other problems and considerations; e.g. non-isothermal transport, is also made correspondingly difficult. In the present note a simplified model is proposed and tested.

The complexity of the basic relations is a direct consequence of the inclusion of the hot bands into the analysis. To avoid this difficulty Edwards [2] considered a single equivalent band and used an average or equivalent value for the line width, b_A, or the broadening parameter, $\beta_A = 2\pi b_A/d$, where d is the line spacing. The relation for β_A was determined from the nonoverlapped strong line condition which corresponds to the maximum influence of the line broadening parameter. This procedure may also be used in conjunction with our previous studies.

From equation (A.5) of [1] we have that the total band absorptance, corresponding to the above condition, is given by

$$\frac{A}{F_P + F_R} = (8\beta/\pi)^{\frac{1}{2}} \sum_v (u_v)^{\frac{1}{2}} = (8\beta X/d\pi)^{\frac{1}{2}} \sum_v (S_{0,v})^{\frac{1}{2}} \qquad (1)$$

where S_0 is a characteristic line intensity, X is the pressure pathlength, F is a characteristic band width, the subscripts P and R refer to the P and R branches, and the optical depth $u_v = S_{0,v}X/d$. In terms of an equivalent band we have (cf. equation (B.4) of [3])

$$\frac{A}{F_P + F_R} = (8\beta_A u_v/\pi)^{\frac{1}{2}} = (8\beta_A X/d\pi)^{\frac{1}{2}} (\sum_v S_{0,v})^{\frac{1}{2}}. \qquad (2)$$

Equating equations (1) and (2) yields

$$(\beta_A/\beta)^{\frac{1}{2}} = \sum_v (S_{0,v})^{\frac{1}{2}}/(\sum_v S_{0,v})^{\frac{1}{2}} \qquad (3)$$

which agrees with the result obtained by Edwards [2]. Now, from [3] we have that

$$S_{0,v} = D_1 \alpha_v^{v+1} (hcB_e/kT)^{\frac{1}{2}} \qquad (4)$$

and from [2] or [4]

$$a_v^{v+1} = \frac{8\pi^3 N_T \nu B^2}{3hcQ_{vib}} (1 - e^{-hc\nu/kT})\, e^{-hc\nu(v+\frac{1}{2})/kT} \qquad (5)$$

where B^2 is the square of the electric dipole matrix element, Q_{vib} is the vibrational partition function and N_T is the total number density. Combining the above relations and using the result that B^2 varies like $(v + 1)$ [2, 4] yields

$$\left(\frac{\beta_A}{\beta}\right)^{\frac{1}{2}} = \frac{\sum_{v=0}^{\infty} [(v+1)\, e^{-vhc\nu/kT}]^{\frac{1}{2}}}{[\sum_{v=0}^{\infty} (v+1)\, e^{-vhc\nu/kT}]^{\frac{1}{2}}}. \qquad (6)$$

Now, using the Euler–Maclaurin summation formula and the identities [5]

$$Y = \sum_{v=0}^{\infty} e^{-vhc\nu/kT} = [1 - e^{-hc\nu/kT}]^{-1}$$